Teaching Statement and Teaching Philosophy

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I. Teaching experience

I have taught every year since 2007 to a variety of audience and contents. I have lectured a section of an undergraduate linear algebra course, led recitations for an advanced graduate course in analysis and general relativity, ran small-group review sessions in undergraduate differential geometry, just to list some examples. Furthermore, in the past two years I took on the role of principal teaching assistant for undergraduate courses in general mathematics and differential geometry in my capacity as a postdoctoral research scientist. The experience I gained — through many organizational duties including the design, administration, and evaluation of midterm and final examinations, and through leading weekly large-group (between 70 to 250 students) exercise sessions — allowed me close looks at which teaching and evaluation strategies work and, more importantly, which do not.

What I discuss below is formulated in part from my personal experience and in part from my informal research into how I can avoid the pitfalls that I have observed. In an effort to further refine my teaching strategies, during the coming semester I will participate in the course "An Introduction to Evidence-Based Undergraduate STEM Teaching" offered on the Coursera platform by Vanderbilt University.

II. Teaching philosophy

I will focus my essay here on the teaching of introductory undergraduate courses, which has the largest impact on the general STEM development of the student body. Furthermore, for higher-level undergraduate and graduate courses, our expectations of the students are different, and so different teaching methods can be used. What started me reassessing how classes are taught and evaluated was the frequency at which…

- … a student inquires, "will this be on the exam?"
- … a student demands to see his final exam, believing that there must be a grading error.
- … a student argues for partial credit for a response that is only a laundry list of formulae.
- … a student tells me during recitation, "I just don't know where to start on this problem."
- … a student asks, "how will the exam be curved?"

These indicate a fundamental disconnect between my goals for the students and their perceptions of such. And I realized that it is not only my job as an instructor to tell them what I expect of them, but also to create an environment that encourages them to meet those expectations.

What is it that I expect?

For many classes (especially so-called "service classes") a main goal is for the students to learn the tools and methods that they can use to tackle problems outside of the classroom setting. Thus my expectations for my students are three-fold:

1. They should learn the subject matter, including how to apply it. (Compare to the first two students above.)
2. They should be able to solve complex problems by planning the use of multiple of the tools and methods acquired in the course or previously. (Compare to the third and fourth.)
3. They should be able to convince others of the correctness of their solutions, and be able to identify the flaws in mathematically unsound arguments. (See again the second student.)

If in the syllabus we only mention the topics which will be taught in the class, and if in the homework assignments we only give straightforward problems like "integrate the following expressions by parts", the students would be under the impression that our goal for the course is only the first of the three items. Therefore starting from the writing of the syllabus, to the design of the course and homework questions, and through the style of the examinations, we need to consistently remind the students of all three goals.

Course design

Syllabus It will contain a detailed description of my three goals and my assessment methods (see below), as well as a list of learning objectives grouped by theme. Example themes for a calculus course can be "Rate of change and total change" which would include the goal "relate the instantaneous rate of change and the total change through the fundamental theorem of calculus and mean value theorem"; and "Optimization" which include the goals "finding critical points of a function, determining whether a critical point is a maximizer or a minimizer"; and "Areas and volumes" which will include "use Cavalieri's principle to compare volumes of different shapes, compute volumes of solids of revolution"; and perhaps the theme "Computation" including the as learning goals familiarity with various methods of evaluating derivatives and integrals.

A typical lesson A lesson is not simply a class period; it is a unit during which a group of concepts is introduced.

1. Motivational discussion in the final 5 to 15 minutes of the previous class. This is achieved by having students discuss in small groups a problem or an example related to the concept to be presented in the lesson.
2. Pre-class homework: a reading assignment with "steps" left out and some exercises.
3. A first part of class where the students discuss their work on the pre-class homework, with input from the instructor.
4. A second part of the class where the missing steps from the reading assignment are filled in by the students and/or the instructor.
5. (The final part of the class belongs to the next lesson.)
6. Some homework problems summarizing the skills discussed in class.

Assessment The formative assessments through in-class discussion and through homework exercises are already indicated in the previous paragraph. The post-class homework questions will be corrected: the grader will take an additive approach indicating to the students what they did *correctly* instead of what they did wrong. To encourage participation a token "participation grade" will be given for in-class participation and homework problems, but the grade is independent of the students' understanding of the material.

For summative assessment I prefer a variant of the standards-based method. The students are told that their grades correlate with their demonstrated proficiency (on exams and tests) in each of the themes and goals. In a class with two tests and one final exam, each of the goals listed in the syllabus will appear at least three times in the questions. A student with at least one correct answer for 80% of the goals earn a passing grade, whereas an 'A' requires two correct answers in each goal.

In addition, for homeworks, tests, and exams, I will implement the idea[1] of "tagging". For all the assessments, the students will have available the list of themes and learning goals as seen on the syllabus. An answer is marked "correct" only if the student correctly identifies which of the goals are addressed in the question.

[1] I learned this from Bret Benesh of The College of St. Benedict and St. John's University.

How this solves the problems

Clarity Evidently a standards-based assessment scheme with clearly delineated expectations brings clarity to the grading. Properly enforced, students should no longer need to ask about the content of the exams nor about the potential for grading curves. Furthermore, among the most important contributions a teacher can bring to a course is the narrative: the understanding of the organizing principles of the subject, how various parts interrelate, and how the subject fits into the grander scheme of things. This understanding is sometimes colloquially referred to as "mathematical maturity". The writing of a syllabus organized by themes projects on the students this organization that they may otherwise not see from a linearly and chronologically ordered listing of subjects (as typical of textbook table of contents or syllabus listing topics by week).

A further improvement to clarity is obtained by the replacing the traditional use of "topics" in the syllabus by the use of "learning goals". The learning goals are description of the desired outcome, and identifies the context in which a student must understand and apply a topic. By switching the syllabus from a teacher-centric presentation (for a student, many of the words in a list of topics are jargon that they have not yet learned) to a student-centric one, we empower the students to take charge of their own learning earlier in the semester.

Compartmentalization Students have tendencies to compartmentalize their knowledge; by this I mean more than mere categorization, but in addition the erection of mental walls between the different categories. This happens when a concept is presented in isolation, and is reinforced by homework exercises which tell the students to solve a particular problem using a given method. A symptom of students suffering from compartmentalization is the lack of problem solving skills. They carry the misconception that mathematics is merely a collection of disconnected tools and formulae to be used when directed to do so by the professor. These students can solve individual problems when broken down for them step-by-step like an IKEA furniture manual, but lack the originality to put all their skills together to actually build a table from scratch.

Once the mental walls are raised, they can be difficult to tear down. Thus we should try to prevent compartmentalization as much as possible. This is addressed in part by the organization of the syllabus by themes. By offering a non-linear organization of the subjects, and reinforcing this organization through tagging, we create an alternative categorization to help the students better see the big picture. Similarly, the motivational discussion and the pre-class homework serve to activate the students' existing knowledge so that new concepts are not learned in isolation but build upon previously learned topics. Yet another way to prevent compartmentalization is in the design of homework exercises. The pre-class exercises serve to activate the students' knowledge and challenge their preconceptions, and so should be broken up into small steps so the students can reason it out using existing knowledge. The after-class exercises should be broader and more open ended, with questions that do not necessarily admit one unique correct answer. Moreover, they should not be limited to the scope of that day's discussions, but should exhibit interplay between all the concepts learned up to then.

Communication Reading mathematics is different from reading a novel, and requires active engagement. To train this the pre-class reading assignments are "interactive": steps are omitted from the discussions and the students are asked to fill in the blanks. Understanding is further promoted by exercise questions that ask for (counter)examples. In writing mathematics one needs to be clear, convincing, and consistent. All these can be trained through peer instruction. By discussing problems and questioning each other's solutions the students learn to critically evaluate mathematical arguments which will lead to better mathematical writing.

Cognition and meta-cognition The "tagging" activity during assessments serves two main purposes. It helps the student know what they know, and it helps the student find out how they know. In regards to the former, by asking the students to place an activity among the themes of the course reinforces a categorization of knowledge that is more helpful to problem solving. Furthermore, it helps them keep track which topics that they have already learned, and coupled with the standards-based assessment scheme encourages students to focus on studying those subjects they have yet to master (by design a student who has already demonstrated proficiency in one subject area twice will not need to do so a third time). This allows them to be more efficient and more productive overall, and also encourages the development of a good self-directed study habit. Relative to the latter, by identifying which skills they used in solving a problem, they reinforce the connection between problem solving and the abstract concepts, through the process usually called meta-cognition. This often leads to better retention of knowledge and improved ability to tackle complex problems.

Another trick to improve (meta-)cognition is through the additive grading system (instead of subtractive) of assignments and exams. By pointing things out to the students we reinforce them, and it is more worthwhile to reinforce correct solutions than incorrect ones. In addition, the process of revising an answer, in particular the discovery of why a wrong answer is wrong, is one of the communication skills we want to train the students in.

III. Courses

Given my specialty and experience, I am confident that I can teach the following courses well.

Introductory undergraduate level: Calculus; linear algebra; differential geometry of curves and surfaces.

Advanced undergraduate level: Real and Fourier analysis; partial differential equations; differential and Riemannian geometry; pseudo-Riemannian geometry and general relativity.

Graduate level: PDEs with focus on nonlinear wave equations; topics in relativity.

In addition I am interested also in introducing mathematical thinking to non-STEM students. Some possible topics of such a course include: Euclidean and projective geometry; discrete mathematics and algorithms; mathematical puzzles.